WESTFIELD PUBLIC SCHOOLS

Westfield, New Jersey

Office of Instruction

Course of Study

ALGEBRA II / ALGEBRA II ADVANCED / ALGEBRA II HONORS

School ..... Westfield High School
Department ..... Mathematics
Length of Course ..... Full year
Grade Levels 10, 11
Prerequisite…Geometry / Geometry Advanced / Geometry Honors[1]
Date .....

I. RATIONALE, DESCRIPTION AND PURPOSE

Students extend their understanding of linear, exponential and quadratic functions, introduced in previous courses, to the study of polynomial, radical, rational and trigonometric functions. Mathematical modeling is used to represent real-world situations and solve problems. Statistical reasoning is employed to evaluate outcomes of experiments, make inferences and draw conclusions. The inter-disciplinary nature of mathematics is emphasized by applications to science and technology.

The three levels of Algebra II cover the same core curriculum. The courses are differentiated by the inclusion of extra topics, pacing and rigor. Algebra II differs from Algebra II Advanced in that there is increased opportunity for review and support in the Algebra II course. Honors Algebra II differs from Algebra II Advanced in that topics are covered in more depth, with a focus on theoretical foundations in the Algebra II Honors course.

The Algebra II course is intended for students who require a more methodical and investigative approach to learning mathematics. Algebra II Advanced is intended for students with a solid algebraic and geometric background. Algebra II Honors is intended for independent learners with an outstanding mathematics background and excellent algebraic and geometric skills.

II. OBJECTIVES

This curriculum fulfills Westfield Board of Education expectations for student achievement. Course objectives are aligned with the New Jersey Student Learning Standards for Mathematics, English Language Arts, Science, Technology, and 21st Century Life and Careers.

[1] See Program of Studies for additional information.

Students:

A. Identify and use arithmetic and geometric sequences and series
NJ Student Learning Standards for Mathematics A-SSE, F-IF, F-BF, F-LE
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

B. Extend knowledge of numbers and operations to the complex number system
NJ Student Learning Standards for Mathematics N-CN
NJ Student Learning Standards for Science P5

C. Solve quadratic equations; graph and use quadratic functions to model and solve quadratic problems
NJ Student Learning Standards for Mathematics N-Q, N-CN, A-REI, F-BF, G-GPE, S-ID
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

D. Solve polynomial equations; graph and use polynomial functions to model and solve polynomial problems
NJ Student Learning Standards for Mathematics N-CN, A-SSE, A-APR, A-REI, F-IF, F-BF
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

E. Solve rational equations; graph and use rational functions to model and solve rational problems
NJ Student Learning Standards for Mathematics A-SSE, A-APR, A-CED, A-REI, F-IF, F-BF
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

F. Solve radical equations; graph and use radical functions to model and solve radical problems
NJ Student Learning Standards for Mathematics N-Q, N-RN, A-REI, F-IF, F-BF
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

G. Solve exponential and logarithmic equations; graph and use exponential and logarithmic functions to model and solve exponential and logarithmic problems
NJ Student Learning Standards for Mathematics N-Q, A-SSE, A-CED, A-REI, F-IF, F-BF, F-LE, S-ID
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

H. Graph and use trigonometric functions to model and solve trigonometric problems
NJ Student Learning Standards for Mathematics N-Q, F-IF, F-BF, F-TF
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

I. Apply principles of experimental design
NJ Student Learning Standards for Mathematics S-IC
NJ Student Learning Standards for English Language Arts A.W1, A.SL2, A.SL3, A.SL4
NJ Student Learning Standards for Science P1, P2, P3, P4, P5, P6, P7, P8
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

J. Recognize and use normal distributions
NJ Student Learning Standards for Mathematics S-ID, S-IC
NJ Student Learning Standards for English Language Arts A.W1, A.SL2, A.SL4
NJ Student Learning Standards for Science P2, P4, P5, P7, P8
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

K. Develop practices and dispositions that lead to mathematical proficiency.
NJ Student Learning Standards for Mathematics SMP1, SMP2, SMP3, SMP4, SMP5, SMP6, SMP7, SMP8
NJ Student Learning Standards for English Language Arts A.R7, A.R10, A.W1, A.SL1, A.SL2, A.SL3, A.SL4, A.SL5
NJ Student Learning Standards for Science P1, P2, P3, P4, P5, P6, P7, P8
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

III. CONTENT, SCOPE AND SEQUENCE

The importance of mathematics in the development of all civilizations and cultures and its relevance to students' success regardless of career path is addressed throughout the secondary mathematics program. Emphasis is placed on the development of critical-thinking and problem-solving skills, particularly through the use of everyday contexts and real-world applications. *(See Appendix VI for pacing)*

A. Review of foundational algebraic concepts[2]
   1. Linear equations, inequalities and systems
   2. Functions
   3. Exponential expressions and equations
   4. Quadratic expressions and equations
   5. Radical expressions and equations
   6. Rational expressions

B. Sequences and series
   1. Recursively defined sequences
      a. Develop and apply recursive formulas for sequences
      b. Generate sequences from recursive formulas
      c. Relate recursive formulas of arithmetic sequences to explicit formulas and linear equations
      d. Relate recursive formulas of geometric sequences to explicit equations and exponential growth and decay functions
   2. Series[3]
      a. Write recursive and explicit formulas for terms of arithmetic and geometric series
      b. Find formulas for partial sums of series and sums of infinite series

C. Construction and transformation of functions
   1. Function concepts
      a. Identify domain and range
      b. Use function notation
      c. Perform operations on functions
      d. Create composite functions algebraically and graphically

[2] Algebra II only
[3] Algebra II Honors

2. Graphs of functions
    a. Develop the parent graph of quadratic functions, highlighting intercepts and extrema
    b. Develop the parent graphs of cubic, square root, cube root, absolute value, step, and piecewise functions
    c. Sketch transformations of parent graphs created by dilations, reflections, and translations
    d. Identify the intervals on which a function is increasing or decreasing
    e. Describe any symmetries of a graph
    f. Recognize even and odd functions from their graphs and/or their equations
    g. Describe the end behavior of a graph
    h. Write the equation of a function given characteristics of its graph
3. Applications of functions
    a. Identify the appropriate domain given the context of a problem
    b. Calculate and interpret the average rate of change of a function presented symbolically or as a table
    c. Estimate the average rate of change of a function from a graph

D. Quadratic functions and complex numbers
1. The imaginary unit and the complex number system
    a. Perform operations with complex numbers
    b. Identify and find conjugates of complex numbers
2. Solutions to quadratic equations
    a. Solve over the real number system by factoring, the square root property, completing the square, and the quadratic formula
    b. Solve over the complex number system by finding non-real solutions as conjugate pairs
3. Graphs of quadratic functions
    a. Make connections between standard, factored, and vertex forms
    b. Graph and analyze quadratic functions using the vertex, axis of symmetry, intercepts, and extremes
4. Quadratic functions to model and solve mathematical and real-world problems (*e.g.,* projectile motion, optimization)

E. Higher-order polynomial functions
1. The structure of polynomials
    a. Define a polynomial and identify examples and non-examples
    b. Define the parts of a polynomial (*e.g.*, leading coefficient, term, constant)
2. Polynomial operations
    a. Add, subtract, and multiply polynomials of any degree
    b. Understand why polynomials are closed under addition, subtraction, and multiplication[4]

[4] Algebra II Advanced, Algebra II Honors

3. The binomial expansion and Pascal's Triangle[5]
    a. Define binomial expansion
    b. Expand $(x + y)^n$ for a positive integer n
    c. Examine the relationship between coefficients of a binomial expansion and the rows of Pascal's Triangle
4. Polynomial factoring
    a. Factor over the real number system
    b. Factor over the complex number system
    c. Factor special cases
    d. Identify zeros of polynomials
5. The relationship between a polynomial equation and its graph
    a. Use the zeroes and intercepts of a polynomial to sketch its graph
    b. Analyze a graph to identify key features (*e.g.,* zeroes, intercepts, intervals where it is increasing/decreasing, extreme values, local extrema, end behavior, degree)
6. Polynomial division
    a. Connect division of polynomials to division of integers
    b. Divide polynomials using long division
    c. Divide polynomials using synthetic division
    d. Know and apply the Remainder and Factor Theorems
7. The Fundamental Theorem of Algebra
    a. Extend the Fundamental Theorem of Algebra to polynomials of higher degree
    b. Analyze the graph of a polynomial function to determine the number of real and complex roots
8. Polynomial equations
    a. Recognize the relationship between zeroes/roots and solutions
    b. Use the Rational Roots Theorem to identify possible rational zeroes
    c. Examine a polynomial's graph, use synthetic division and/or factoring to find all solutions to a polynomial equation
    d. Write polynomial functions that possess given roots/zeroes
9. Polynomial inequalities
    a. Solve quadratic and rational inequalities via interval testing
    b. Analyze the graph of a polynomial to determine the solution(s) of an inequality

F. Exponential and logarithmic models
1. Exponential expressions
    a. Simplify expressions containing integer exponents
    b. Simplify expressions containing rational exponents
2. Exponential functions
    a. Sketch exponential functions by transformations and study the effect on intercepts and end behavior
    b. Relate the domain and range of exponential functions to their graphs
    c. Describe the intervals where the graph of the function is increasing or decreasing
    d. Find the equation of an exponential function given characteristics of its graph
    e. Use the change of base formula to solve an exponential equation
    f. Develop the meaning of base *e*

[5] Algebra II Advanced, Algebra II Honors

g. Sketch natural exponential functions by transformations and study the effect on intercepts and end behavior
h. Model and solve exponential growth, populations growth, and compound and continuous interest problems

3. Inverse relations and functions
   a. Distinguish between inverse relations and inverse functions
   b. Determine if a function is one-to-one
   c. Find the inverse of a function
   d. Use composition of functions to verify that two functions are inverses[6]
   e. Determine the values of an inverse given a graph or table
   f. Restrict the domain of a function that is not invertible in order to make it invertible[7]
4. Logarithmic functions
   a. Define logarithms as the inverse of exponents
   b. Convert between exponential and logarithmic form
   c. Use properties of logarithms to expand and condense logarithmic expressions
   d. Sketch logarithmic functions by transformations and study the impact on intercepts and end behavior
   e. Relate the domain and range of logarithmic functions to their graphs
   f. Describe the intervals where the graph of a function is increasing or decreasing
   g. Solve logarithmic equations by condensing and equating arguments or converting to exponential form
   h. Solve exponential equations by using logarithms
5. Exponential and logarithmic functions to model and solve mathematical and real-world problems

G. Rational functions
1. Operations on rational expressions
   a. Define rational expressions and identify restrictions
   b. Add, subtract, multiply and divide rational expressions and express result in simplest form
   c. Understand why rational expressions are closed under addition, subtraction, and multiplication[8]
   d. Simplify complex fractions
2. Rational equations and inequalities
   a. Solve rational equations by using a least common denominator
   b. Identify and explain extraneous solutions
   c. Solve rational inequalities by analyzing values over certain intervals that are included and/or excluded in the solution
3. Graphs of rational functions
   a. Define a rational function
   b. Graph a rational function by highlighting key concepts: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity

[6] Algebra II Advanced, Algebra II Honors
[7] Algebra II Honors
[8] Algebra II Advanced, Algebra II Honors

c. Explain the relationship between the domain of a rational function and its graph
4. Inverse of a rational function
5. Rational functions to model and solve mathematical and real-world problems

H. Trigonometric functions
1. Trigonometric ratios
   a. Revisit right triangle definitions for the sine, cosine and tangent
   b. Define positive angles in standard position, reference angles and quadrant location
   c. Use the unit circle to extend the definitions of the sine, cosine and tangent to any angle
   d. Define a new unit of angle measure, *radian*, in terms of arc length on the unit circle
2. Periodic behavior and sinusoidal functions
   a. Define periodic functions
   b. Graph parent functions of the sinusoidals $y = \cos x$ and $y = \sin x$
   c. Apply knowledge of transformations to the graphs of sinusoidal functions
3. Periodic behavior and trigonometric functions[9]
   a. Apply knowledge of transformations to the graphs of trigonometric functions, studying the impact of period, amplitude, phase shift and midline
   b. Write sine and cosine equations based on periodic observances, graphs, or stated data
   c. Use trigonometric functions as mathematical models to make predictions and interpret real-world situations
4. Fundamental trigonometric identities[10]
   a. Prove the quotient identity $\tan x = \sin x/\cos x$ using definitions derived from the unit circle
   b. Prove the Pythagorean identity $\sin^2 x + \cos^2 x = 1$
   c. Use identities and the quadrant of the angle to find the value of $\sin x$, $\cos x$, or $\tan x$
   d. Prove the sum and difference formulas for sine, cosine and tangent[11]
   e. Use the sum and difference formulas to solve problems involving acute angles[12]

I. Foundations of statistical analysis
1. Univariate data
   a. Calculate and interpret measures of center and spread
   b. Graph univariate data using histograms, box plots, dot plots and stem plots
2. Normal distribution of data sets
   a. Fit normal data sets to the standard normal curve
   b. Use tables and/or the calculator to determine standard z-scores and estimate percentile rank

3. Sample surveys, experiments, and observational studies
   a. Recognize their purposes and differences
   b. Explain how randomness applies to each situation
   c. Decide when cause-and-effect relationships can be established

[9] Algebra II Honors
[10] Algebra II Honors
[11] Algebra II Honors
[12] Algebra II Advanced, Algebra II Honors

IV. INSTRUCTIONAL TECHNIQUES

A variety of instructional approaches is employed to engage all students in the learning process and accommodate differences in readiness levels, interests and learning styles. Typical teaching techniques include, but are not limited to, the following:

A. Teacher-directed, whole-group instruction, and modeling of procedures
B. Mini-lessons or individualized instruction for reinforcement or re-teaching of concepts
C. Guided investigations/explorations
D. Problem-based learning
E. Modeling with manipulatives
F. Flexible grouping
G. Differentiated tasks
H. Spiral review
I. Independent practice
J. Use of technology
K. Integration of mathematics with other disciplines.

V. EVALUATION

Multiple techniques are employed to assess student understanding of mathematical concepts, skills, and thinking processes. These may include, but are not limited to, the following:

A. Written tests and quizzes, including baseline and benchmark assessments
B. Cumulative tests
C. Standardized tests
D. Electronic data-gathering and tasks
E. Homework
F. Independent or group projects
G. Presentations.

VI. PROFESSIONAL DEVELOPMENT

The following recommended activities support this curriculum:

A. Opportunities to learn from and share ideas about teaching and learning mathematics with colleagues through meetings and peer observations
B. Collaboration with colleagues and department supervisor to discuss and reflect upon unit plans, homework, and assessment practices
C. Planning time to develop and discuss the results of implementing differentiated lessons and incorporating technology to enhance student learning
D. Attendance at workshops, conferences and courses that focus on relevant mathematics content, pedagogy, alternate assessment techniques, or technology.

APPENDIX I

New Jersey Student Learning Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

SMP1 – Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

SMP2 – Reason abstractly and quantitatively.
Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

SMP3 – Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

SMP4 – Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

SMP5 – Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

SMP6 – Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

SMP7 – Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

SMP8 – Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

New Jersey Student Learning Standards for Mathematical Content

The Real Number System N-RN

2. Rewrite expressions involving radicals and ration exponents using the properties of exponents.

Quantities N-Q

Reason quantitatively and use units to solve problems.

2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System N-CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with *a* and *b* real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Arithmetic with Polynomials and Rational Expressions A-APR

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number *a*, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[13]

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations A-CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Reasoning with Equations and Inequalities A-REI

Understand solving equations as a process of reasoning and explain the reasoning.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Represent and solve equations and inequalities graphically.

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

[13] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Interpreting Functions F-IF

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*

Linear and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

4. For exponential models, express as a logarithm the solution to $a\,b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Prove and apply trigonometric identities.

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant.

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable.

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Making Inferences and Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.

2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

(+) Denotes additional mathematics that students should learn in order to take advanced courses.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX II

New Jersey Student Learning Standards for English Language Arts

College and Career Readiness Anchor Standards for Reading:

NJSLSA.R7 – Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.

NJSLSA.R10 – Read and comprehend complex literary and informational texts independently and proficiently.

College and Career Readiness Anchor Standard for Writing:

NJSLSA.W1 – Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

College and Career Readiness Anchor Standards for Speaking and Listening:

NJSLSA.SL1 – Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

NJSLSA.SL2 – Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.

NJSLSA.SL3 – Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

NJSLSA.SL4 – Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.

NJSLSA.SL5 – Make strategic use of digital and visual displays of data to express information and enhance understanding of presentations.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX III

New Jersey Student Learning Standards for Science / Next Generation Science Standards: Science and Engineering Practices

Practice 1 – Asking questions and defining problems
Practice 2 – Developing and using models
Practice 3 – Planning and carrying out investigations
Practice 4 – Analyzing and interpreting data
Practice 5 – Using mathematics and computational thinking
Practice 6 – Constructing explanations and designing solutions
Practice 7 – Engaging in argument from evidence
Practice 8 – Obtaining, evaluating, and communicating information

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX IV

New Jersey Student Learning Standards for Technology

NJSLS 8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX V

New Jersey Student Learning Standards for 21st Century Life and Careers

NJSLS Career Ready Practices: These practices outline the skills that all individuals need to have to be truly adaptable, reflective, and proactive in life and careers. These are researched practices that are essential to career readiness.

NJSLS 9.1 Personal Financial Literacy: This standard outlines the important fiscal knowledge, habits, and skills that must be mastered in order for students to make informed decisions about personal finance. Financial literacy is an integral component of a student's college and career readiness, enabling students to achieve fulfilling, financially-secure, and successful careers.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX VI

Instructional Resources and Pacing Guides

Instructional resource for Algebra II & Algebra II Advanced: *Discovering Advanced Algebra,* Murdock et al, Key Curriculum Press (2004).

Suggested pacing*:*

Unit	# teaching days	
Course	Algebra II	Algebra II Advanced
Patterns and recursion	10	1
Linear models and systems	8	15
Functions, relations and transformations	22	25
Exponential, power and logarithmic functions	42	36
Quadratic and other polynomial functions	30	51
Rational functions	24	15
Trigonometric functions	26	18
Data and statistics	5	9

Instructional resource for Algebra II Honors: *Algebra & Trigonometry with Analytic Geometry*, Swokowski & Cole, Brooks/Cole (2002).

Suggested pacing:

Unit	# teaching days
Developing mathematical proficiencies	8
Fundamental concepts of algebra	21
Equations and inequalities	19
Functions and graphs	22
Polynomials and rational functions	17
Exponential and logarithmic functions	21
Trigonometric functions	30
Sequences and series	15
Statistics	10

WESTFIELD PUBLIC SCHOOLS

Westfield, New Jersey

Office of Instruction

Course of Study

PRECALCULUS ADVANCED (6231)

School……Westfield High School
Department…….Mathematics
Length of Course…….….Full year
Credit……... 5
Grade Level …….11, 12
Prerequisite……..Algebra II Advanced[1]
Date……..……..

I. RATIONALE, DESCRIPTION AND PURPOSE

Students extend and strengthen their knowledge of elementary functions while studying advanced mathematics topics such as trigonometry, conic sections, logarithmic and exponential functions, matrices, probability, sequences and series. Mathematical modeling is used to represent real-world situations and solve problems. The interdisciplinary nature of mathematics is emphasized by applications to science and technology.

The three levels of Precalculus cover much of the same core curriculum. The courses are differentiated by the inclusion of extra topics, pacing and rigor. Precalculus Advanced differs from Precalculus in that there is increased opportunity for review and support in the Precalculus course. Precalculus Advanced differs from Precalculus Honors in that topics are covered in more depth, with a focus on theoretical foundations, in the Precalculus Honors course. The Precalculus Advanced course is intended for students with a solid algebraic and geometric background.

II. OBJECTIVES

This curriculum fulfills Westfield Board of Education expectation for student achievement. Course objectives are aligned with the New Jersey Student Learning Standards for Mathematics, English Language Arts, Science, Technology, and 21st Century Life and Careers.

[1] See Program of Studies for additional information.

Students:

A. Derive the Law of Sines, Law of Cosines and area formulas and use to solve problems involving non-right triangles
NJ Student Learning Standards for Mathematics N-Q, A-SSE, A-CED, A-REI, F-TF, G-SRT
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

B. Extend knowledge of the relationship among arcs, angles and radian measure
NJ Student Learning Standards for Mathematics N-Q, F-BF, F-TF
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

C. Graph the six trigonometric functions with various transformations and use them to model and solve mathematical and real-world problems
NJ Student Learning Standards for Mathematics N-Q, A-CED, A-REI, F-IF, F-BF, F-TF
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

D. Define and apply the inverses of the trigonometric functions
NJ Student Learning Standards for Mathematics N-Q, F-BF, A-REI
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

E. Extend knowledge of identities to include sum and difference, double angle, half angle, co-function, and odd-even identities
NJ Student Learning Standards for Mathematics N-Q, F-TF
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

F. Use basic identities to simplify and verify complex identities
NJ Student Learning Standards for Mathematics A-SSE, F-TF
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

G. Solve trigonometric equations using identities as necessary
NJ Student Learning Standards for Mathematics A-REI, F-TF
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

H. Use trigonometric functions to solve mathematical and real-world problems, including those involving right triangles, oblique triangles, and areas
NJ Student Learning Standards for Mathematics N-Q, A-CED, F-BF, G-SRT
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

I. Identify, write, graph and analyze the equations of conic sections
NJ Student Learning Standards for Mathematics N-Q, A-CED, A-REI, F-BF, G-GPE
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

J. Define the fundamentals of matrix arithmetic and apply matrices to model real-world situations
NJ Student Learning Standards for Mathematics N-Q, N-VM
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

K. Extend knowledge of exponential and logarithmic functions to solve mathematical and real-world problems
NJ Student Learning Standards for Mathematics N-Q, A-SSE, A-CED, A-REI, F-BF, F-LE
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

L. Identify, manipulate and evaluate various types of sequences and series
NJ Student Learning Standards for Mathematics N-Q, A-SSE, F-BF
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

M. Calculate and use expected values to solve problems and evaluate outcomes of decisions
NJ Student Learning Standards for Mathematics N-Q, F-BF, S-MD
NJ Student Learning Standards for Science P5, P8
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

N. Develop practices and dispositions that lead to mathematical proficiency
NJ Student Learning Standards for Mathematics SMP1, SMP2, SMP3, SMP4, SMP5, SMP6, SMP7, SMP8
NJ Student Learning Standards for English Language Arts A.R7, A.R10, A.W1, A.SL1, A.SL2, A.SL3, A.SL4, A.SL5
NJ Student Learning Standards for Science P1-P8
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

III. CONTENT, SCOPE AND SEQUENCE

The importance of mathematics in the development of all civilizations and cultures and its relevance to students' success regardless of career path is addressed throughout the secondary mathematics program. Emphasis is placed on the development of critical-thinking and problem-solving skills, particularly through the use of everyday contexts and real-world applications. *(See Appendix VI for pacing)*

A. Functions
1. Identification of a relation as a function given its equation or graph
2. Combinations and composition of functions
3. Inverse relations and invertible functions
4. Domain, range, combinations and composition of functions
5. Identification of a function as odd, even or neither
6. Analysis of the behavior of a function including boundedness, extremea, and end behavior

B. Triangle trigonometry
1. Applications of trigonometry to find missing sides and angles of right triangles
2. Area of a triangle using the appropriate formula, including Hero's formula and the formula for SAS

3. Derivation of Law of Sines and Law of Cosines
4. Applications of Law of Sines and Law of Cosines to find missing sides and angles of a triangle
5. Real-world applications with triangle trigonometry

C. Trigonometric functions
1. Conversion between degree and radian measures
2. Review of evaluating trigonometric functions
3. Graphs of the six trigonometric functions with transformations, including an analysis of period, amplitude, phase displacement, sinusoidal axis, domain and range
4. Definition and evaluation of the six inverse trigonometric functions with range restrictions

D. Trigonometric equations and applications
1. Solution and application of simple trigonometric equations
2. Creation and application of equations for various sine and cosine curves
3. Use of trigonometric functions to model periodic behavior
4. Derivation of co-function and odd-even identities
5. Derivation and application of sum, difference and double-angle identities
6. Application of half-angle identities
7. Simplification of trigonometric expressions and proof of trigonometric identities
8. Use of trigonometric identities and technology to solve more difficult trigonometric equations and inequalities

E. Conics
1. Graphs of circles, ellipses, hyperbolas and parabolas in the rectangular coordinate plane
2. Identification of a conic given in general form; rewriting the equation in standard form
3. Determination of the equation of a conic given defining characteristics
4. Real-world problems involving conics

F. Matrices
1. Solutions of systems of equations in more than two variables
2. Definition of the concept of a matrix and use of matrices to represent data
3. Addition, subtraction, scalar multiplication and multiplication of matrices
4. Definition, evaluation and application of determinants of 2x2 and 3x3 matrices
5. Inverses of 2x2 and 3x3 matrices
6. Use of matrices to solve linear systems

G. Logarithmic and exponential functions
1. Review of simplification of a logarithmic expression, including common logarithms and natural logarithms, through converting to exponential form or the change of base formula
2. Review of use of transformations to graph exponential functions and identify domain and range
3. Review of use of the product, quotient and power properties to expand and condense logarithmic expressions
4. Review of solving various types of logarithm and exponential equations
5. Applications of logarithmic and exponential functions

H. Sequences and series
   1. Review of definitions of arithmetic and geometric sequences
   2. Sequences rewritten in general form
   3. Expansion and evaluation of a sum given in sigma notation
   4. Use of sigma notation to rewrite a given sum
   5. Definitions for and evaluation of arithmetic and geometric series
   6. Limits of infinite sequences
   7. Evaluation of infinite series and definition of converging/diverging series
   8. Applications of sequences and series

I. Probability
   1. Determination of the values of a random variable and corresponding probabilities
   2. Calculation of a probability distribution function/table from data
   3. Graphical display of a probability distribution function
   4. Calculation of the expected value (mean) of a random variable
   5. Use of probability to evaluate outcomes of decisions
      a. Expected "payoff" to determine fairness
      b. Random number generator to simulate real-world scenarios
      c. Analysis of strategies and results

IV. INSTRUCTIONAL TECHNIQUES

A variety of instructional approaches is employed to engage all students in the learning process and accommodate differences in readiness levels, interests and learning styles. Typical teaching techniques may include, but are not limited to, the following:

A. Teacher-directed, whole-group instruction, and modeling of procedures
B. Mini-lessons or individualized instruction for reinforcement of re-teaching of concepts
C. Guided investigations/explorations
D. Problem-based learning
E. Modeling with manipulatives
F. Flexible grouping
G. Differentiated tasks
H. Spiral review
I. Independent practice
J. Use of technology
K. Integration of mathematics with other disciplines.

V. EVALUATION

Multiple techniques are employed to assess student understanding of mathematical concepts, skills, and thinking processes. These may include, but are not limited to, the following:

A. Written tests and quizzes, including baseline and benchmark assessments
B. Cumulative tests
C. Standardized tests
D. Electronic data-gathering and tasks
E. Homework
F. Independent or group projects
G. Presentations.

VI. PROFESSIONAL DEVELOPMENT

The following recommended activities support this curriculum:

A. Opportunities to learn from and share ideas about teaching and learning mathematics with colleagues through meetings and peer observations
B. Collaboration with colleagues and department supervisor to discuss and reflect upon unit plans, homework, and assessment practices
C. Planning time to develop and discuss results of implementing differentiated lessons and incorporating technology to enhance student learning
D. Attendance at workshops, conferences and courses that focus on relevant mathematics content, pedagogy, alternate assessment techniques, or technology.

APPENDIX I

New Jersey Student Learning Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

SMP1 – Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

SMP2 – Reason abstractly and quantitatively.
Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

SMP3 – Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a

logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

SMP4 – Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

SMP5 – Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

SMP6 – Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are

careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

SMP7 – Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

SMP8 – Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

New Jersey Student Learning Standards for Mathematical Content

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Vector and Matrix Quantities N-VM

Perform operations on matrices and use matrices in applications.

1. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

2. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
3. (+) Add, subtract, and multiply matrices of appropriate dimensions.
4. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
5. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
7. (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Seeing Structure in Expressions A-SSE

Write expressions in equivalent forms to solve problems.

4. Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Creating Equations A-CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

Reasoning with Equations and Inequalities A-REI

Solve systems of equations.

6. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Represent and solve equations and inequalities graphically.

12. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
13. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Interpreting Functions F-IF

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
   d. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

Build new functions from existing functions.

4. Find inverse functions.
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Use the inverse relationship between exponents and logarithms to solve problems involving exponents and logarithms.

Linear and Exponential Models F-LE

Construct and compare linear and exponential models and solve problems.

4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Trigonometric Functions F-TF

Model periodic phenomena with trigonometric functions.

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Similarity, Right Triangles, and Trigonometry G-SRT

Define trigonometric ratios and solve problems involving right triangles

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles

9. (+) Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Law of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section.

3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems.

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and*

calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

Use probability to evaluate outcomes of decisions.

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
   b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

(+) Denotes additional mathematics that students should learn in order to take advanced courses.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX II

New Jersey Student Learning Standards for English Language Arts

College and Career Readiness Anchor Standards for Reading:
NJSLSA.R7 – Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.

NJSLSA.R10 – Read and comprehend complex literary and informational texts independently and proficiently.

College and Career Readiness Anchor Standards for Writing:
NJSLSA.W1 – Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

College and Career Readiness Anchor Standards for Speaking and Listening:
NJSLSA.SL1 – Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

NJSLSA.SL2 – Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.

NJSLSA.SL3 – Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

NJSLSA.SL4 – Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.

NJSLSA.SL5 – Make strategic use of digital and visual displays of data to express information and enhance understanding of presentations.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX III

New Jersey Student Learning Standards for Science / Next Generation Science Standards: Science and Engineering Practices

Practice 1 – Asking questions and defining problems
Practice 2 – Developing and using models
Practice 3 – Planning and carrying out investigations
Practice 4 – Analyzing and interpreting data
Practice 5 – Using mathematics and computational thinking
Practice 6 – Constructing explanations and designing solutions
Practice 7 – Engaging in argument from evidence
Practice 8 – Obtaining, evaluating, and communicating information

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX IV

New Jersey Student Learning Standards for Technology

NJSLS 8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX V

New Jersey Student Learning Standards for 21st Century Life and Careers

NJSLS Career Ready Practices: These practices outline the skills that all individuals need to have to be truly adaptable, reflective, and proactive in life and careers. These are researched practices that are essential to career readiness.

NJSLS 9.1 Personal Financial Literacy: This standard outlines the important fiscal knowledge, habits, and skills that must be mastered in order for students to make informed decisions about personal finance. Financial literacy is an integral component of a student's

college and career readiness, enabling students to achieve fulfilling, financially-secure, and successful careers.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX VI
Instructional Resources and Pacing Guide

Instructional resource: *Precalculus Graphical, Numerical, Algebraic*, Demana et al, Pearson (2015).

Suggested pacing*:*

Unit	# teaching days
Functions and graphs	15
Polynomial, power and rational functions	15
Exponential, logistic, and logarithmic functions	16
Trigonometric functions	27
Analytic trigonometry	27
Systems and matrices	15
Analytic geometry in two and three dimensions	12
Discrete mathematics	9
Statistics and probability	8
Introduction to calculus: limits, derivatives, and integrals	10

WESTFIELD PUBLIC SCHOOLS

Westfield, New Jersey

Office of Instruction

Course of Study

PRECALCULUS HONORS (6233)

School…….Westfield High School
Department……..…….Mathematics
Length of Course…….……..Full year
Credit ……5
Grade Level ……11
Prerequisite……...Algebra II Honors[1]
Date…….

I. RATIONALE, DESCRIPTION AND PURPOSE

Students extend and strengthen their knowledge of elementary functions while studying advanced mathematics and introductory calculus topics such as trigonometry, conic sections, polar coordinates, vectors, matrices, probability, sequences and series, limits and continuity. Mathematical modeling is used to represent real-world situations and solve problems. The interdisciplinary nature of mathematics is emphasized by applications to science and technology.

The three levels of Precalculus cover much of the same core curriculum. The courses are differentiated by the inclusion of extra topics, pacing and rigor. Precalculus Honors differs from Precalculus Advanced in that topics are covered in more depth, with a focus on theoretical foundations. Precalculus Honors is intended for independent learners with an outstanding mathematics background and excellent algebraic and geometric skills.

II. OBJECTIVES

This curriculum fulfills Westfield Board of Education expectation for student achievement. Course objectives are aligned with the New Jersey Student Learning Standards for Mathematics, English Language Arts, Science, Technology, and 21st Century Life and Careers.

Students:

A. Identify, graph and analyze the equations of the conic sections, including reference to linear transformations and rotations with an emphasis on the end behavior of the function
NJ Student Learning Standards for Mathematics A-CED, A-REI, F-BF, G-GPE
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

[1] See Program of Studies for additional information.

B. Extend knowledge of the relationship among arcs, angles and radian measure and find the area of sectors of circles
NJ Student Learning Standards for Mathematics N-Q, F-BF, F-TF, G-CO
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

C. Graph the six trigonometric functions with various transformations and use to model and solve mathematical and real-world problems
NJ Student Learning Standards for Mathematics N-Q, A-CED, A-REI, F-IF, F-BF, F-TF
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

D. Define, graph and apply the inverses of the trigonometric functions
NJ Student Learning Standards for Mathematics N-Q, A-REI, F-BF
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

E. Extend knowledge of identities to include sum and difference, double angle, half angle, co-function, and odd-even identities
NJ Student Learning Standards for Mathematics N-Q, F-TF
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

F. Utilize basic identities to simplify and verify complex identities
NJ Student Learning Standards for Mathematics A-SSE, F-TF
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

G. Solve trigonometric equations and inequalities, using identities as necessary
NJ Student Learning Standards for Mathematics A-REI, F-TF
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

H. Utilize trigonometric functions to solve mathematical and real-world problems, including those involving right and oblique triangles, areas and vectors
NJ Student Learning Standards for Mathematics N-Q, A-CED, F-BF, G-SRT
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

I. Define polar coordinates, relate to complex numbers, and perform operations on complex numbers using polar form
NJ Student Learning Standards for Mathematics N-CN
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

J. Graph simple and complex curves using polar coordinates
NJ Student Learning Standards for Mathematics N-Q, F-BF, A-REI
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

K. Represent vectors algebraically and geometrically, perform basic operations and solve mathematical and real-world problems involving vectors
NJ Student Learning Standards for Mathematics N-Q, N-VM
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

L. Investigate the relationship between vectors and parametric equations
NJ Student Learning Standards for Mathematics F-BF, N-VM
NJ Student Learning Standards for Science P3, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

M. Define the fundamentals of matrix arithmetic and apply matrices to various types of mathematical problems, including partial fraction decomposition
NJ Student Learning Standards for Mathematics N-VM, F-BF
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

N. Solve systems of linear and quadratic equations algebraically, graphically and using matrices
NJ Student Learning Standards for Mathematics N-VM, A-CED, A-REI, F-BF
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

O. Calculate and use expected values to solve problems and evaluate outcomes of decisions
NJ Student Learning Standards for Mathematics N-Q, F-BF, S-MD
NJ Student Learning Standards for Science P5, P8
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

P. Identify, manipulate and evaluate the limits of various types of sequences and series, and test for convergence of infinite series
NJ Student Learning Standards for Mathematics A-SSE, A-CED, F-IF
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

Q. Evaluate the limits of various types of functions and analyze the continuity of the functions
NJ Student Learning Standards for Mathematics F-BF, F-IF
NJ Student Learning Standards for Science 5.1, 5.3
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

R. Apply the principles of continuity to various types of functions
NJ Student Learning Standards for Mathematics F-IF
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

S. Calculate and analyze the instantaneous rate of change of a function
NJ Student Learning Standards for Mathematics F-IF, G-GPE
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

T. Develop practices and dispositions that lead to mathematical proficiency.
NJ Student Learning Standards for Mathematics SMP1, SMP2, SMP3, SMP4, SMP5, SMP6, SMP7, SMP8
NJ Student Learning Standards for English Language Arts A.R7, A.R10, A.W1, A.SL1, A.SL2, A.SL3, A.SL4, A.SL5
NJ Student Learning Standards for Science P1-P8
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

III. CONTENT, SCOPE AND SEQUENCE

The importance of mathematics in the development of all civilizations and cultures and its relevance to students' success regardless of career path is addressed throughout the secondary mathematics program. Emphasis is placed on the development of critical-thinking and problem-solving skills, particularly through the use of everyday contexts and real-world applications. *(See Appendix VI for pacing)*

A. Conics
   1. Equations of circles and points of intersection of circles and lines
   2. Equations and graphs of ellipses, hyperbolas and parabolas
   3. Real-world problems involving conics
   4. Solution of systems of second degree equations
   5. Investigation of rotated and degenerate conics

B. Trigonometric functions
   1. Arc length and area of sectors of circles, with reference to the definition of radian measure
   2. Applications involving apparent size
   3. Review of evaluating trigonometric functions
   4. Graphs of the six trigonometric functions with transformations, including an analysis of period, amplitude, domain and range
   5. Definition and evaluation of the six inverse trigonometric functions with range restrictions
   6. Graphs and applications of inverse trig functions

C. Trigonometric equations and applications
   1. Solution and application of trigonometric equations
   2. Creation and application of equations for sine and cosine curves
   3. Use of trigonometric functions to model periodic behavior
   4. Derivation of co-function and odd-even identities
   5. Derivation and application of sum and difference of angle identities
   6. Derivation and application of double-angle and half-angle identities
   7. Simplification of trigonometric expressions and proof of trigonometric identities
   8. Application of trigonometric identities and technology to solve complex trigonometric equations and inequalities

D. Triangle trigonometry
   1. Review of the use of trigonometry to find missing sides and angles of right triangles
   2. Area of a triangle using the appropriate formula, including Hero's formula and $A = \frac{1}{2} ab \sin C$
   3. Review of Law of Sines and Law of Cosines to find missing sides and angles of a triangle
   4. Applications with trigonometry

E. Polar coordinates and complex numbers
   1. Relationship between polar and rectangular coordinates and conversion between forms
   2. Conversion between polar and rectangular equations
   3. Graphs of polar equations and identification of types of polar curves
   4. Use of polar forms of complex numbers to find products and quotients of the numbers
   5. Derivation and application of DeMoivre's Theorem for powers/roots of complex numbers

F. Vectors
   1. Geometric representation of vectors and basic operations
   2. Algebraic representation of vectors in component form and basic operations
   3. Correlation between the vector representations and polar-rectangular conversions
   4. Application of vectors to various real-world scenarios
   5. Use of vector and parametric equations to describe motion in a plane
   6. Investigation of parametric equations and their graphs, including conversion to Cartesian equations

G. Matrices
   1. Review of solving systems of first- and second-degree equations in two variables graphically and algebraically
   2. Solutions of systems of equations in three variables
   3. Definition of the concept of a matrix and use of matrices to represent data
   4. Addition, subtraction, scalar multiplication and multiplication of matrices
   5. Investigation of properties of matrix operations
   6. Definition, evaluation and application of the determinants of 2x2 and 3x3 matrices, including vector applications
   7. Inverses of 2x2 and 3x3 matrices
   8. Use of matrices to solve linear systems (*e.g.*, Cramer's Rule, row-echelon operations, technology)
   9. Application of linear systems to real-world scenarios, including partial fraction decomposition

H. Probability
   1. Definition and application of random variables
   2. Creation and graphs of probability distribution functions
   3. Calculation of expected value and solution of related problems
   4. Application of the concepts of expected value to decision making

I. Sequences and series
   1. Review of arithmetic and geometric sequences and series
   2. Limits of infinite sequences
   3. Evaluation of infinite series and definition of converging/diverging series
   4. Tests to determine infinite series convergence/divergence
   5. Definition and evaluation of the interval of convergence for a power series

J. Limits of functions
   1. Review of rational functions and their graphs, including asymptotes and end behavior models
   2. Limit of a function as *x* approaches a finite value
   3. Limit of a function as *x* approaches positive or negative infinity
   4. Graphical interpretation of limits
   5. Interpretation of limits in real-world applications
   6. Development of theorems involving limits
   7. Definition and analysis of the continuity of a function

K. Derivatives and instantaneous rates of change
   1. Comparisons between the average rate of change with instantaneous rates of change
   2. Definition and application of derivatives to functions
   3. Derivation of the Power rule of differentiation
   4. Derivation of derivatives of sine and cosine functions
   5. Derivation and application of Product and Quotient Rules of differentiation

IV. INSTRUCTIONAL TECHNIQUES

A variety of instructional approaches is employed to engage all students in the learning process and accommodate differences in readiness levels, interests and learning styles. Typical teaching techniques may include, but are not limited to, the following:

A. Teacher-directed, whole-group instruction, and modeling of procedures
B. Mini-lessons or individualized instruction for reinforcement or re-teaching of concepts
C. Guided investigations/explorations
D. Problem-based learning
E. Modeling with manipulatives
F. Flexible grouping
G. Differentiated tasks
H. Spiral review
I. Independent practice
J. Use of technology
K. Integration of mathematics with other disciplines.

V. EVALUATION

Multiple techniques are employed to assess student understanding of mathematical concepts, skills and thinking processes. These may include, but are not limited to, the following:

A. Written tests and quizzes, including baseline and benchmark assessments
B. Cumulative tests
C. Standardized tests
D. Electronic data-gathering and tasks
E. Homework
F. Independent or group projects
G. Presentations.

VI. PROFESSIONAL DEVELOPMENT

The following recommended activities support this curriculum:

A. Opportunities to learn from and share ideas about teaching and learning mathematics with colleagues through meetings and peer observations
B. Collaboration with colleagues and department supervisor to discuss and reflect upon unit plans, homework, and assessment practices
C. Planning time to develop and discuss the results of implementing differentiated lessons and incorporating technology to enhance student learning
D. Attendance at workshops, conferences and courses that focus on relevant mathematics content, pedagogy, alternate assessment techniques, or technology.

APPENDIX I

New Jersey Student Learning Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

SMP1 – Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

SMP2 – Reason abstractly and quantitatively.
Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

SMP3 – Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a

logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

SMP4 – Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

SMP5 – Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

SMP6 – Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are

careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

SMP7 – Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

SMP8 – Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

New Jersey Student Learning Standards for Mathematical Content

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System N-CN

Perform arithmetic operations with complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.*
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Vector and Matrix Quantities N-VM

Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
   b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

12. (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Seeing Structure in Expressions A-SSE

Write expressions in equivalent forms to solve problems.

4. Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Creating Equations A-CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Reasoning with Equations and Inequalities A-REI

Solve systems of equations.

6. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Represent and solve equations and inequalities graphically.

12. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
13. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Interpreting Functions F-IF

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
   d. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

Build new functions from existing functions.

4. Find inverse functions.
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Trigonometric Functions F-TF

Model periodic phenomena with trigonometric functions.

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Circles G-C

Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section.

3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems.

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions.

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
   b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

(+) Denotes additional mathematics that students should learn in order to take advanced courses.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX II

New Jersey Student Learning Standards for English Language Arts

College and Career Readiness Anchor Standards for Reading:
NJSLSA.R7 – Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.

NJSLSA.R10 – Read and comprehend complex literary and informational texts independently and proficiently.

College and Career Readiness Anchor Standards for Writing:
NJSLSA.W1 – Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

College and Career Readiness Anchor Standards for Speaking and Listening:
NJSLSA.SL1 – Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

NJSLSA.SL2 – Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.

NJSLSA.SL3 – Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

NJSLSA.SL4 – Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.

NJSLSA.SL5 – Make strategic use of digital and visual displays of data to express information and enhance understanding of presentations.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX III

New Jersey Student Learning Standards for Science / Next Generation Science Standards: Science and Engineering Practices

Practice 1 – Asking questions and defining problems
Practice 2 – Developing and using models
Practice 3 – Planning and carrying out investigations
Practice 4 – Analyzing and interpreting data
Practice 5 – Using mathematics and computational thinking
Practice 6 – Constructing explanations and designing solutions
Practice 7 – Engaging in argument from evidence
Practice 8 – Obtaining, evaluating, and communicating information

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/

APPENDIX IV

New Jersey Student Learning Standards for Technology

NJSLS 8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX V

New Jersey Student Learning Standards for 21st Century Life and Careers

NJSLS Career Ready Practices: These practices outline the skills that all individuals need to have to be truly adaptable, reflective, and proactive in life and careers. These are researched practices that are essential to career readiness.

NJSLS 9.1 Personal Financial Literacy: This standard outlines the important fiscal knowledge, habits, and skills that must be mastered in order for students to make informed decisions about personal finance. Financial literacy is an integral component of a student's college and career readiness, enabling students to achieve fulfilling, financially-secure, and successful careers.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX VI

Instructional Resources and Pacing Guide

Instructional resource: *Precalculus Graphical, Numerical, Algebraic*, Demana et al, Pearson (2015).

Suggested pacing*:*

Unit	# teaching days
Polynomial, power and rational functions and graphs	5
Exponential, logistic, and logarithmic functions	5
Trigonometric functions	17
Analytic trigonometry	24
Applications of trigonometry	27
Systems and matrices	16
Analytic geometry in two and three dimensions	15
Discrete mathematics and probability	8
Limits	17
Derivatives	12
Calculus sequences and series	8

WESTFIELD PUBLIC SCHOOLS

Westfield, New Jersey

Office of Instruction

Course of Study

PRECALCULUS (6441)

School …….. Westfield High School
Department ……Mathematics
Length of Course ……..Full year
Credit ……..…….…… 5
Grade Levels ……11, 12
Prerequisite…….Algebra II
Date ……

I. RATIONALE, DESCRIPTION AND PURPOSE

Students extend and strengthen their knowledge of elementary functions while studying advanced mathematics topics such as trigonometry, logarithms, matrices, conics, probability, sequences and series. Mathematical modeling is used to represent real-world situations and solve problems. The interdisciplinary nature of mathematics is emphasized by applications to science and technology.

The Precalculus course is intended for students who require a more methodical and investigative approach to learning mathematics. Alongside the core curriculum is increased opportunity for review and support.

II. OBJECTIVES

This curriculum fulfills Westfield Board of Education expectations for student achievement. Course objectives are aligned with the New Jersey Student Learning Standards for Mathematics, English Language Arts, Science, Technology, and 21st Century Life and Careers.

Students:

A. Extend knowledge of the relationship among arcs, angles and radian measure
NJ Student Learning Standards for Mathematics N-Q, F-BF, F-TF
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

B. Identify and apply the six trigonometric ratios
NJ Student Learning Standards for Mathematics N-Q, F-TF
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

C. Use trigonometric identities to simplify expressions, determine exact values, and solve equations
NJ Student Learning Standards for Mathematics N-Q, A-CED, A-SSE, F-TF, G-SRT
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

D. Derive the Law of Sines, Law of Cosines and area formulas and use to solve problems involving non-right triangles
NJ Student Learning Standards for Mathematics N-Q, A-CED, A-SSE, A-REI, F-TF, G-SRT
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

E. Use parent graphs and transformations of the six trigonometric functions to model and solve problems
NJ Student Learning Standards for Mathematics N-Q, F-IF, F-BF, F-TF, A-CED, A-SSE, A-REI
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

F. Use trigonometry to solve mathematical and real-world problems, including those involving right triangles, oblique triangles, and areas
NJ Student Learning Standards for Mathematics N-Q, A-CED, F-BF, G-SRT
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

G. Identify, manipulate, evaluate, and solve problems involving sequences and series
NJ Student Learning Standards for Mathematics N-Q, A-SSE, F-BF
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

H. Extend knowledge of exponential and logarithmic functions to solve mathematical and real-world problems
NJ Student Learning Standards for Mathematics N-Q, A-SSE, A-CED, A-REI, F-BF, F-LE
NJ Student Learning Standards for Science P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

I. Define the fundamentals of matrix arithmetic and apply matrices to model real-world situations
NJ Student Learning Standards for Mathematics N-Q, N-VM
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

J. Identify, write, graph, and analyze the equations of conic sections
NJ Student Learning Standards for Mathematics N-Q, A-CED, A-REI, F-BF, G-GPE
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

K. Use rules for probability and conditional probability to solve problems
NJ Student Learning Standards for Mathematics S-MD, S-CP
NJ Student Learning Standards for Science P2, P5
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

L. Calculate and use expected values to solve problems and evaluate outcomes of decisions
NJ Student Learning Standards for Mathematics N-Q, F-BF, S-MD
NJ Student Learning Standards for Science P5, P8
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

M. Develop practices and dispositions that lead to mathematical proficiency.
NJ Student Learning Standards for Mathematics SMP1, SMP2, SMP3, SMP4, SMP5, SMP6, SMP7, SMP8
NJ Student Learning Standards for English Language Arts A.R7, A.R10, A.W1, A.SL1, A.SL2, A.SL3, A.SL4, A.SL5
NJ Student Learning Standards for Science P1 – P8
NJ Student Learning Standards for Technology 8.1
NJ Student Learning Standards for 21st Century Life and Careers 9.1

III. CONTENT, SCOPE AND SEQUENCE

The importance of mathematics in the development of all civilizations and cultures and its relevance to students' success regardless of career path is addressed throughout the secondary mathematics program. Emphasis is placed on the development of critical-thinking and problem-solving skills, particularly through the use of everyday contexts and real-world applications. *(See Appendix VI for pacing)*

A. Review of elementary function concepts
1. Domain and range
2. Inverse relations and functions
3. Odd and even functions
4. Parent graphs and transformations
5. Piecewise functions
6. Composition of functions

B. Trigonometry
1. Right triangles and the unit circle
   a. Conversion between degree and radian measures
   b. The six trigonometric functions
   c. The unit circle
2. Trigonometric identities
   a. Reciprocal, Pythagorean, and quotient identities
   b. Sum, difference and double-angle formulas for sine, cosine, and tangent
   c. Determination of exact values using identities
   d. Simplification and verification of expressions using identities
3. Trigonometric equations
   a. Application of inverse trigonometric functions
   b. Use of identities to solve trigonometric equations
   c. Solutions in degrees and radians over a restricted domain

4. Non-right triangles
    a. Law of Sines and Law of Cosines
        1) Missing side lengths and angle measures
        2) Investigation of the ambiguous case
    b. Area formulas
        1) Hero's Formula
        2) $A = \frac{1}{2} ab \sin C$
    c. Applications with non-right triangles
5. Graphical representation of the six trigonometric functions
    a. Characteristics of each type of graph
    b. Transformations of parent graphs
    c. Applications of trigonometric functions

C. Sequences and series
1. Review of arithmetic and geometric sequences and series
    a. Formulas for the nth term
    b. Summation formulas
2. Partial sums of arithmetic and geometric series

D. Logarithmic and exponential functions
1. Exponential equations written in logarithmic form and vice versa
2. Common logarithms and natural logarithms
3. Properties of logarithms
4. Solutions of logarithmic and exponential equations
5. Applications of logarithmic and exponential functions

E. Polynomial and rational functions
1. Graphical and algebraic analysis of polynomial and rational functions
2. Simplification of complex rational expressions
3. Identification and classification of discontinuities graphically and algebraically
4. Creation of graphs given characteristics (intercepts, intervals where function is increasing/decreasing, relative minimums/maximums, periodicity, end behavior)

F. Matrices
1. Matrices to model given situations and display data
2. Operations on matrices (addition, subtraction, multiplication, scalar multiplication)
3. Representation of a system of linear equations as a matrix equation
4. Inverse of a matrix to solve a system of linear equations
5. Cramer's Rule to solve a system of linear equations

G. Conics
1. Standard form to identify and write equations of circles, parabolas, ellipses and hyperbolas
2. Equation of a circle, parabola, ellipse and hyperbola given defining characteristics
3. Graphs of circles, parabolas, ellipses and hyperbolas in the rectangular coordinate plane
4. Real-world applications of conics

H. Probability
   1. Basic concepts
      a. Applications of the Addition and Multiplication Rules
      b. Application of probability to make fair decisions and analyze strategies
   2. Conditional probability and independence
      a. Identification of independent events
      b. Recognition and calculation of conditional probability
      c. Relationship between the concepts of conditional probability and independence to everyday situations
   3. Expected values
      a. Definition of random variable
      b. Values of a random variable and corresponding probabilities
      c. Probability distribution function/table
      d. Graphical display of a probability distribution function
      e. Calculation of expected value (mean) of a random variable

IV. INSTRUCTIONAL TECHNIQUES

A variety of instructional approaches is employed to engage all students in the learning process and accommodate differences in readiness levels, interests and learning styles. Typical teaching techniques may include, but are not limited to, the following:

A. Teacher-directed, whole-group instruction, and modeling of procedures
B. Mini-lessons or individualized instruction for reinforcement or re-teaching of concepts
C. Guided investigations/explorations
D. Problem/based learning
E. Modeling with manipulatives
F. Flexible grouping
G. Differentiated tasks
H. Spiral review
I. Independent practice
J. Use of technology
K. Integration of mathematics with other disciplines.

V. EVALUATION

Multiple techniques are employed to assess student understanding of mathematical concepts, skills, and thinking processes. These may include, but are not limited to, the following:

A. Written tests and quizzes, including baseline and benchmark assessments
B. Cumulative tests
C. Standardized tests
D. Electronic data-gathering and tasks
E. Homework
F. Independent or group projects
G. Presentations.

VI. PROFESSIONAL DEVELOPMENT

The following recommended activities support this curriculum:

A. Opportunities to learn from and share ideas about teaching and learning mathematics with colleagues through meetings and peer observations
B. Collaboration with colleagues and department supervisor to discuss and reflect upon unit plans, homework, and assessment practices
C. Planning time to develop and discuss results of implementing differentiated lessons and incorporating technology to enhance student learning
D. Attendance at workshops, conferences and courses that focus on relevant mathematics content, pedagogy, alternate assessment techniques, or technology.

APPENDIX I

New Jersey Student Learning Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

SMP1 – Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

SMP2 – Reason abstractly and quantitatively.
Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

SMP3 – Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical

progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

SMP4 – Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

SMP5 – Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

SMP6 – Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with

quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

SMP7 – Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

SMP8 – Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

New Jersey Student Learning Standards for Mathematical Content

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Vector and Matrix Quantities N-VM

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in

a game are doubled.

8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Creating Equations A-CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R*.*

Reasoning with Equations and Inequalities A-REI

Understand solving equations as a process of reasoning and explain the reasoning.

2. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Represent and solve equations and inequalities graphically.

11. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Interpreting Functions F-IF

Understand the concept of a function and use function notation.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   f. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.
   b. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   c. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   d. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.

Model periodic phenomena with trigonometric functions.

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities.

15. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant.
16. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Similarity, Right Triangles, and Trigonometry G-SRT

Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles.

9. (+) Derive the formula $A = \frac{1}{2}\, ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Expressing Geometric Properties with Equations G-GPE

Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).

Making Inferences and Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Conditional Probability and the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as $P(A \text{ and } B)/P(B)$, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems.

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions.

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
   b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high- deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

(+) Denotes additional mathematics that students should learn in order to take advanced courses.

The entire standards document may be viewed at [http://www.state.nj.us/education/cccs/.](http://www.state.nj.us/education/cccs/)

APPENDIX II

New Jersey Student Learning Standards for English Language Arts

College and Career Readiness Anchor Standards for Reading:

NJSLSA.R7 – Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.

NJSLSA.R10 – Read and comprehend complex literary and informational texts independently and proficiently.

College and Career Readiness Anchor Standards for Writing:

NJSLSA.W1 – Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

College and Career Readiness Anchor Standards for Speaking and Listening:
NJSLSA.SL1 – Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

NJSLSA.SL2 – Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.

NJSLSA.SL3 – Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

NJSLSA.SL4 – Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.

NJSLSA.SL5 – Make strategic use of digital and visual displays of data to express information and enhance understanding of presentations.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/.

APPENDIX III

New Jersey Student Learning Standards for Science / Next Generation Science Standards: Science and Engineering Practices

Practice 1 – Asking questions and defining problems
Practice 2 – Developing and using models
Practice 3 – Planning and carrying out investigations
Practice 4 – Analyzing and interpreting data
Practice 5 – Using mathematics and computational thinking
Practice 6 – Constructing explanations and designing solutions
Practice 7 – Engaging in argument from evidence
Practice 8 – Obtaining, evaluating, and communicating information

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/

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APPENDIX IV

New Jersey Student Learning Standards for Technology

NJSLS 8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/

APPENDIX V

New Jersey Student Learning Standards for 21st Century Life and Careers

NJSLS Career Ready Practices: These practices outline the skills that all individuals need to have to be truly adaptable, reflective, and proactive in life and careers. These are researched practices that are essential to career readiness.

NJSLS 9.1 Personal Financial Literacy: This standard outlines the important fiscal knowledge, habits, and skills that must be mastered in order for students to make informed decisions about personal finance. Financial literacy is an integral component of a student's college and career readiness, enabling students to achieve fulfilling, financially-secure, and successful careers.

The entire standards document may be viewed at http://www.state.nj.us/education/cccs/

APPENDIX VI

Instructional Resources and Pacing Guide

Instructional resource: *Precalculus with Trigonometry*, Foerster, Key Curriculum Press (2007).

Suggested pacing*:*

Unit	# teaching days
Functions and mathematical models	15
Periodic function and right triangle problems	13
Applications of trigonometric and circular functions	12
Trigonometric function properties, identities, and parametric functions	17
Properties of combined sinusoids	7
Triangle trigonometry	12
Properties of elementary functions	15
Probability and functions of a random variable	10
Matrix transformations and fractal figures	10
Analytic geometry of conic sections and quadric surfaces	19
Sequences and series	7
Polynomial and rational functions, limits, and derivatives	17